

# Einstein's $R^{\hat{0}\hat{0}}$ equation for nonrelativistic sources derived from Einstein's inertial motion and the Newtonian law for relative acceleration

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With Einstein's inertial motion (freefalling and nonrotating relative to gyroscopes), geodesics for nonrelativistic particles can intersect repeatedly, allowing one to compute the space-time curvature  $R^{\hat{0}\hat{0}}$  exactly. Einstein's  $R^{\hat{0}\hat{0}}$  for strong gravitational fields and for relativistic source-matter is *identical* with the Newtonian expression for the relative radial acceleration of neighbouring freefalling test-particles, spherically averaged. — Einstein's field equations follow from Newtonian experiments, local Lorentz-covariance, and energy-momentum conservation combined with the Bianchi identity.

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Up to now, a rigorous derivation of Einstein's field equations for general relativity has been lacking: Wald [1] writes “a clue is provided”, “the correspondence suggests the field equation”. Weinberg [2] takes the “weak static limit”, makes a “guess”, and argues with “number of derivatives”. Misner, Thorne, and Wheeler [3] give “Six Routes to Einstein's field equations”, among which they recommend (1) “model geometrodynamics after electrodynamics”, (2) “take the variational principle with only a scalar linear in second derivatives of the metric and no higher derivatives”.

In contrast, we give a rigorous derivation of Einstein's field equations for general relativity.

The crucial input is Einstein's concept of inertial motion (freefalling and nonrotating relative to comoving gyroscopes): the worldlines of freefalling nonrelativistic test-particles, geodesics, can intersect repeatedly. Two stones released one after the other from rest at the North Pole freefalling into a vertical well through the center of the Earth to the South Pole and back through the center of the Earth to the North Pole: Einstein's geodesics cross repeatedly, violating the spacetime analogue of Euclid's axiom of parallels, evidence that space-time is curved.

In our decisive first step, we prove that the exact space-time curvature encoded in  $R^{\hat{0}}_{\hat{0}}(P)$  (curvature-side of Einstein's equation) is fully *determined* by measuring test-particles which are *quasi-static* (or non-relativistic) relative to an observer with worldline through  $P$  and  $\bar{u}_{\text{obs}}(P) = \bar{e}_{\hat{0}}(P)$ . It is superfluous to re-measure  $R^{\hat{0}}_{\hat{0}}$  with relativistic test-particles. — Further, we prove that  $R^{\hat{0}}_{\hat{0}}$  is *identical* with the Newtonian expression for the relative radial acceleration of neighboring freefalling nonrelativistic test-particles, spherically averaged, for gravitational fields of arbitrary strength and for arbitrary (relativistic) source-matter. — Hats over indices denote Local Ortho-Normal Bases (LONB) following Misner, Thorne, and Wheeler, and we use their sign conventions [3].

The *explicit* expression for Einstein's curvature  $R^0_0$  in general coordinates in terms of Christoffel connection coefficients  $(\Gamma^\alpha_\beta)_\gamma \equiv \Gamma^\alpha_{\beta\gamma}$  has 106 terms, utterly unin-

structive. Newtonian relative acceleration in general Lagrangian 3-coordinates (e.g. comoving with the wind) has the same number of 106 unconstructive terms.

The expressions for Einstein's  $R^{\hat{0}}_{\hat{0}}(P)$  and the Newtonian relative acceleration are extremely simple and *explicitly identical* with the following choices: (1) We work with Local Ortho-Normal Bases (LONBs) in Cartan's method. (2) We use a *primary observer* (non-inertial or inertial) with worldline through  $P$ , with  $\bar{u}_{\text{obs}} = \bar{e}_{\hat{0}}$ , and with his spatial LONBs  $\bar{e}_{\hat{i}}$  along his worldline. (3) We use the primary observer's spacetime slicing  $\Sigma_t$  by radial 4-geodesics starting Lorentz-orthogonal to his worldline, and  $t \equiv$  time measured on his worldline. (4) Most crucial: for measuring relative accelerations of neighbouring test-particles, we need *auxiliary observers* with *LONBs radially parallel* (at a given time) to the primary observer's LONBs (to avoid unnecessary extra terms). Therefore the Ricci connection coefficients for radial displacements from the primary observer's worldline vanish,

$$\begin{aligned} &\text{auxiliary observers} \\ &\text{with radially parallel LONBs at given time} \\ &\Leftrightarrow [(\omega^{\hat{a}}_{\hat{b}})_{\hat{i}}]_{r=0} = 0, \end{aligned} \quad (1)$$

for  $(a, b, \dots) =$  spacetime indices,  $(i, j, \dots) =$  3-space indices. Eq. (1) is a special case of Newtonian observers at relative rest at a given time. (5) We use Riemann normal 3-coordinates centered on the primary observer on the slices of fixed time:  $r$ -coordinate lines are radial geodesics,  $r \equiv$  geodesic radial distance,  $(\theta, \phi)_P \equiv$  starting angles of radial geodesics to  $P$ , and  $(x, y, z)_P$  with the standard connection to spherical coordinates.

In this first paper, we treat only *inertial primary observers*. Hence the Ricci connection coefficients for displacements along the primary observer's worldline vanish,

$$\text{inertial primary observer} \Leftrightarrow [(\omega^{\hat{a}}_{\hat{b}})_{\hat{0}}]_{r=0} = 0. \quad (2)$$

With Eqs. (1, 2), *all* Ricci connection coefficients vanish on the worldline of the primary inertial observer. Our auxiliary observers cannot be inertial, unless spacetime is flat.

The result: the expression for Einstein's  $R_{\hat{0}}^{\hat{0}}$  in terms of quasi-static (or non-relativistic) test-particles, for gravitational fields of arbitrary strength, and for arbitrary (relativistic) source-matter, is *exactly and explicitly identical* with the Newtonian expression, and this expression is *exactly linear* in the gravitational field,

inertial primary observer, radially parallel LONBs:

$$\Leftrightarrow R_{\hat{0}}^{\hat{0}} = \text{div } \vec{E}_g = \text{div } \vec{g}. \quad (3)$$

In our exact operational definition in arbitrary (3+1)-spacetimes, the gravito-electric field  $\vec{E}_g = \vec{g}$  is the acceleration of *quasi-static* (or non-relativistic) freefalling test-particles, measured by the chosen observer. But this  $\vec{E}_g$  remains exactly valid for *relativistic* test-particles in the equations of motion and in curvature calculations.

In a companion paper, we shall show in detail that for *non-inertial observers* and for quasi-static (or non-relativistic) test-particles, (1) the exact explicit expression for Einstein's  $R_{\hat{0}}^{\hat{0}}$  and the 19th-century Newtonian expression for relative acceleration of neighbouring freefalling particles, spherically averaged, are *identical*, if one uses Einstein's *equivalence* of *fictitious forces* and *gravitational forces* ( $\vec{E}_g, \vec{B}_g$ ), which has been demonstrated explicitly in [4], and (2) that the two identical expressions are *nonlinear* in the gravitational fields.

In the second (trivial) step for deriving Einstein's  $R_{\hat{0}}^{\hat{0}}$  equation, we put non-relativistic source-matter on the matter-side of Einstein's equation: it follows that Einstein's  $R_{\hat{0}}^{\hat{0}}$  equation for nonrelativistic source-matter and for gravitational fields of arbitrary strength is *exactly identical* with the Newtonian equation for the relative radial acceleration of neighbouring freefalling test-particles, spherically averaged.

In a third, well-known step, given in textbooks, one derives the general Einstein equations from Einstein's  $R_{\hat{0}}^{\hat{0}}$  equation for nonrelativistic source-matter by using local Lorentz covariance and energy-momentum conservation combined with the Bianchi identity.

These three steps complete our rigorous derivation of Einstein's field equations for general relativity. — Additional results in [4].

The tools needed in this paper are: (1) our exact operational definition of the gravito-electric field  $\vec{E}_g$ , (2) the Ricci connection coefficients for a Lorentz boost of LONBs under a displacement in time,  $(\omega^{\hat{i}}_{\hat{0}})_{\hat{0}}$ , and (3) our identity  $E_{\hat{i}}^{(g)} = -(\omega^{\hat{i}}_{\hat{0}})_{\hat{0}}$ .

The gravito-electric field  $\vec{E}_g$  measured by any local observer (with his LONBs along his worldline) is given by our exact and general operational definition in arbitrary (3+1)-spacetimes, Eq. (4), which is probably new. — In contrast to the literature, we use no perturbation theory on a background geometry, no weak gravitational fields. —  $\vec{E}_g$  is defined as the measured acceleration of *quasistatic* freefalling test-particles analogous to the op-

erational definition of the ordinary electric field, where we replace the particle's charge by its rest mass  $m$ ,

$$m^{-1} \frac{d}{dt} p_{\hat{i}} \equiv E_{\hat{i}}^{(g)} \\ \Leftrightarrow \vec{a}_{\text{ff}} = \vec{E}^{(g)} = \vec{g}, \quad (4)$$

for freefalling, quasistatic test-particles.

Local time-intervals  $dt$  are measured on the observer's wristwatch. The measured 3-momentum is  $p_{\hat{i}}$  with respect to the observer's LONB. For a freefalling test-particle, quasistatic relative to the observer, the measured gravitational acceleration relative to the observer is  $\vec{a}_{\text{ff}}^{(\text{quasistatic})} = \vec{g} = \vec{E}_g$ , measured by Galilei.

The LONB-components  $p^{\hat{a}}$  are *directly measurable*. This is in stark contrast to coordinate-basis components  $p^{\alpha}$ , which are not measurable before one has obtained  $g_{\alpha\beta}$  by solving Einstein's equations for the specific problem at hand.

LONBs off the observer's worldline are not needed in Eq. (4), because a particle released from rest (or quasistatic state) will still be on the observer's worldline after an infinitesimal time  $\delta t$ , since  $\delta s \propto (\delta t)^2 \Rightarrow 0$ , while  $\delta v \propto \delta t \neq 0$ .

For a freefalling observer,  $\vec{E}_g = \vec{g}$  is zero on his worldline: Einstein's "happiest thought of my life".

Gravito-electric fields  $\vec{E}_g$  of arbitrary strength can be measured exactly with freefalling test-particles which are *quasistatic* relative to the observer, Eq. (4). But this same measured  $\vec{E}_g$  is *exactly* valid for *relativistic* test-particles in the equations of motion.

The gravito-magnetic field  $\vec{B}_g$  has been postulated by Heaviside in 1893 [5]. Our exact operational definition of  $\vec{B}_g$  is given in [4].

The term "weak gravitational fields" for *local* discussions is often used in textbooks. But "weak gravity" is meaningless locally, because the gravitational field  $\vec{g}$  and the gravitational tidal field  $R_{\hat{0}}^{\hat{0}}$  are not dimensionless.

*Cartan's method* with LONB-connection coefficients is unavoidable for our computation of curvature from measurements by non-inertial observers. But Cartan's LONB method is not taught in almost all graduate programs in general relativity in the USA, and most researchers have never used Cartan's method to solve a problem. Therefore we introduce elements of Cartan's method.

*Ricci's LONB-connection coefficients* are illustrated by an airplane on the shortest path (geodesic) from Zurich to Chicago and the Local Ortho-Normal Bases (LONBs) chosen to be in the directions "East" and "North". These LONBs rotate relative to the geodesic (relative to parallel transport) with a rotation angle  $\delta\alpha$  per measured path length  $\delta s$ , i.e. with the rotation rate  $\omega = (d\alpha/ds)$ .

For infinitesimal displacements  $\delta\vec{D}$  in *any* direction, the rotation angle  $\delta\alpha$  of LONBs is given by a linear map encoded by the Ricci rotation coefficients  $\omega_{\hat{c}}$ ,

$$\delta\alpha = \omega_{\hat{c}} \delta D_{\hat{c}}.$$

The Ricci rotation coefficients are also called *connection* coefficients, because they connect the LONBs at infinitesimally neighboring points by a rotation relative to the infinitesimal geodesic between these points.

Cartan's LONB connection coefficients use displacements in the *coordinates*,

$$\delta\alpha = \omega_\gamma \delta D^\gamma.$$

In three spatial dimensions, the rotation of LONBs relative to the geodesic from  $P$  to  $Q$  must be given by a rotation matrix. For a rotation in the  $(\vec{e}_{\hat{x}}, \vec{e}_{\hat{y}})$ -plane,

$$\begin{pmatrix} \vec{e}_{\hat{x}} \\ \vec{e}_{\hat{y}} \end{pmatrix}_Q = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \vec{e}_{\hat{x}} \\ \vec{e}_{\hat{y}} \end{pmatrix}_P.$$

For infinitesimal displacements, hence infinitesimal rotations (first derivatives in  $\alpha$ ), the rotation matrix is,

$$\begin{pmatrix} \vec{e}_{\hat{x}} \\ \vec{e}_{\hat{y}} \end{pmatrix}_Q = \left[ 1 + \alpha \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \begin{pmatrix} \vec{e}_{\hat{x}} \\ \vec{e}_{\hat{y}} \end{pmatrix}_P.$$

The infinitesimal LONB-rotation matrix  $\delta R_{i\hat{j}}$  is given by the linear map from the infinitesimal coordinate-displacement vector  $D^{\hat{c}}$ ,

$$\delta R_{i\hat{j}} = (\omega_{i\hat{j}})_{\hat{c}} \delta D^{\hat{c}},$$

$$\omega_{1\hat{2}} = -\omega_{\hat{2}1} = \alpha_{1\hat{2}} = \text{rotation angle in } [\hat{1}, \hat{2}] \text{ plane.}$$

The  $(\omega_{i\hat{j}})_{\hat{c}}$  are the Ricci connection coefficients.

In (1+1)-spacetime, the Lorentz transformation of the chosen LONBs relative to a given displacement geodesic is a Lorentz boost  $L^{\hat{a}}_{\hat{b}}$ ,

$$\begin{pmatrix} \vec{e}_{\hat{t}} \\ \vec{e}_{\hat{x}} \end{pmatrix}_Q = \begin{pmatrix} \cosh \chi & \sinh \chi \\ \sinh \chi & \cosh \chi \end{pmatrix} \begin{pmatrix} \vec{e}_{\hat{t}} \\ \vec{e}_{\hat{x}} \end{pmatrix}_P,$$

with  $\tanh \chi \equiv v/c$ , with  $\chi$  called “rapidity”, and  $\chi$  additive for successive Lorentz boosts in the same spatial direction. For infinitesimal displacements, the infinitesimal Lorentz boost  $L^{\hat{a}}_{\hat{b}}$  is,

$$\begin{pmatrix} \vec{e}_{\hat{t}} \\ \vec{e}_{\hat{x}} \end{pmatrix}_Q = \left[ 1 + \chi \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right] \begin{pmatrix} \vec{e}_{\hat{t}} \\ \vec{e}_{\hat{x}} \end{pmatrix}_P.$$

In (3+1)-spacetime, and with two lower indices,  $\omega_{\hat{a}\hat{b}}$  is antisymmetric for Lorentz boosts (and for rotations),

$$\delta L_{\hat{a}\hat{b}} = (\omega_{\hat{a}\hat{b}})_{\hat{c}} \delta D^{\hat{c}}, \quad (\omega_{i\hat{0}})_{\hat{c}} = -(\omega_{\hat{0}i})_{\hat{c}} = (\chi_{i\hat{0}})_{\hat{c}}.$$

For a displacement in observer-time, the *exact* Ricci connection coefficients  $(\omega_{\hat{a}\hat{b}})_{\hat{0}}$  of general relativity can be measured in *quasistatic* experiments. But these Ricci connection coefficients predict the motion of *relativistic* particles with the equations of motion.

Our gravito-electric field  $\vec{E}_g$  is identical with minus the Ricci Lorentz-boost coefficients for a displacement in time,

$$E_i^{(g)} = -(\omega_{i\hat{0}})_{\hat{0}}. \quad (5)$$

The proof: from the point of view of the observer with his LONBs along his worldline, the gravitational acceleration  $g_i = a_i^{(\text{ff particle})}$  of freefalling *quasistatic* test-particles (starting on the observer's worldline) is by definition identical to the *exact* gravitoelectric field  $E_i$  of general relativity, Eq. (4). — But from the point of view of freefalling test-particles, the acceleration of the quasistatic observer with his LONBs is by definition identical to the *exact* Ricci LONB-boost coefficients  $(\omega_{i\hat{0}})_{\hat{0}}$ ,

$$\begin{aligned} E_i^{(g)} &\equiv [(a_i)^{(\text{relat.to obs.})}_{\text{ff particle}}]_{\text{quasistatic}} = g_i \\ &= -[(a_i)^{(\text{relat.to ff})}_{\text{observer}}]_{\text{quasistatic}} \equiv -(\omega_{i\hat{0}})_{\hat{0}}. \end{aligned}$$

Galilei measured exact Ricci connection coefficients of general relativity:  $(\omega_{i\hat{0}})_{\hat{0}} = \delta_{i\hat{z}} (9.1 \text{ m/s}^2)$  for LONBs in directions East, North, vertical.

Our general, exact definition of the gravitomagnetic field,  $\vec{B}_g/2 \equiv \vec{\Omega}_{\text{gyroscope}}^{(\text{relat.to obs.})}$ , is discussed in [4]. The Ricci connection coefficients  $(\omega_{i\hat{j}})_{\hat{0}}$  equal minus the precession rate of gyroscopes (comoving with the observer),  $(\omega_{i\hat{j}})_{\hat{0}} = -\Omega_{i\hat{j}}^{(\text{gyro})} \equiv -\varepsilon_{i\hat{j}\hat{k}} \Omega_{\hat{k}}^{(\text{gyro})}$ . These exact Ricci connection coefficients of general relativity were measured by Foucault in 1853.

In striking contrast, Christoffel connection coefficients (for coordinate bases),  $\Gamma^\alpha_{\beta\gamma} \equiv (\Gamma^\alpha_\beta)_\gamma$ , have no direct physical-geometric meaning, and they cannot be known, until the metric fields  $g_{\mu\nu}(x)$  have been obtained by solving Einstein's equations for a given problem.

We write Christoffel connection coefficients with a bracket: inside the bracket are the coordinate-basis transformation-indices  $(\alpha, \beta)$ , outside the bracket is the coordinate-displacement index  $\gamma$ .

For *curvature* computations there are two methods, (1) the standard method with coordinate bases and Christoffel connections  $(\Gamma^\alpha_\beta)_\gamma$ , (2) Cartan's method with Local Ortho-Normal Bases and LONB-connections  $(\omega^{\hat{a}}_{\hat{b}})_\gamma$ .

For a primary non-inertial observer, *Cartan's* method is strongly preferred, because a radially parallel LONB-vector  $\vec{e}_{\hat{0}}(P)$  off the primary observer's worldline, which is highly convenient for measuring relative radial acceleration, does not point in the same direction as the natural coordinate-basis vector  $\vec{e}_0(P) = \partial_t$  for a rotating or non-freefalling observer.

Cartan's curvature equation gives the Riemann curvature 2-form  $\mathcal{R}^{\hat{a}}_{\hat{b}}$  with 2-form components  $(\mathcal{R}^{\hat{a}}_{\hat{b}})_{\gamma\delta}$  [6, 7]. 2-form components are antisymmetric covariant components in a coordinate basis, denoted by Greek letters. — For an *inertial* primary observer and with our *LONBs radially parallel*, all of Cartan's LONB connection coefficients  $(\omega^{\hat{a}}_{\hat{b}})_\gamma$  vanish on the worldline of the primary observer, Eqs. (1, 2). Therefore, in Cartan's curvature equation, the term bilinear in the connection, the wedge product (antisymmetric in the suppressed coordinate-basis displacement-indices)  $[\omega^{\hat{a}}_{\hat{c}} \wedge \omega^{\hat{c}}_{\hat{b}}]$  vanishes. Hence

Cartan's curvature 2-form  $\mathcal{R}^{\hat{a}}_{\hat{b}}$  is equal to the exterior derivative  $d$  of the LONB-connection 1-form  $\omega^{\hat{a}}_{\hat{b}}$  in notation free of form-components,

$$\mathcal{R}^{\hat{a}}_{\hat{b}} = d\omega^{\hat{a}}_{\hat{b}}, \quad (6)$$

where  $d$  denotes the antisymmetric ordinary partial derivative, and  $(\hat{a}, \hat{b})$  are the Lorentz-transformation indices of the LONBs.

Writing explicitly the antisymmetric 2-form-component indices  $[\mu, \nu]$  (plaquette indices) on the left-hand-side and the antisymmetric pair of derivative-index and displacement-index on the right-hand side, Eq. (6) reads,

$$(\mathcal{R}^{\hat{a}}_{\hat{b}})_{\mu\nu} = (d\omega^{\hat{a}}_{\hat{b}})_{\mu\nu} \equiv \partial_\mu (\omega^{\hat{a}}_{\hat{b}})_\nu - [\mu \leftrightarrow \nu]. \quad (7)$$

An instructive elementary derivation of the curvature equations (6, 7) for 2-space is given in [4].

Eqs. (6, 7) give our crucial curvature result for general relativity:

inertial primary observer,

radially parallel LONBs:

$$\begin{aligned} R^{\hat{0}}_{\hat{0}} &= (\mathcal{R}^{\hat{0}}_{\hat{i}})_{\hat{0}\hat{i}} = \\ &= -\partial_{\hat{i}}(\omega^{\hat{0}}_{\hat{i}})_{\hat{0}} = \text{div } \vec{E}_g = \partial_r <a_r^{\text{ff}}>_{\text{ang. average}} \end{aligned} \quad (8)$$

The last expression states that Einstein's *exact*  $R^{\hat{0}}_{\hat{0}}$  curvature is *identical* with the Newtonian relative acceleration of freefalling test-particles, spherically averaged for gravitational fields of arbitrary strength and for arbitrary source-matter (e.g. relativistic). — It is superfluous to re-measure or recompute  $R^{\hat{0}}_{\hat{0}}$  with relativistic test-particles.

The *second step*, the derivation of Einstein's  $R^{\hat{0}}_{\hat{0}}$  equation for non-relativistic sources is now trivial: we write the sources on the right-hand-side of the equation,

inertial primary observer, radially parallel LONBs,  
nonrelativistic sources:

$$\begin{aligned} R^{\hat{0}}_{\hat{0}} & \quad \text{Einstein exact} \\ &= \partial_r <a_r^{\text{ff}}>_{\text{ang. average}} \quad \text{Newton} \\ &= \text{div } \vec{E}_g \quad \text{Gauss} \\ &= -4\pi G_N \rho_{\text{mass}}. \end{aligned} \quad (9)$$

It has been often emphasized that a fundamental difference between general relativity and Newtonian physics is the *non-linearity* of Einstein's equations versus the *linearity* of the Newton-Gauss equation  $\text{div } \vec{E}_g = -4\pi G_N \rho_{\text{mass}}$ . Nothing could be farther from the truth: We have given the proof that Einstein's *exact*  $R^{\hat{0}}_{\hat{0}}(P)$  and  $\text{div } \vec{E}_g(P)$  of Newton-Gauss are explicitly *identical* and *linear* in the gravitational field  $\vec{g} = \vec{E}_g$  for an *inertial primary observer* in  $P$  with  $\bar{u}_{\text{obs}} = \bar{e}_{\hat{0}}$ , if one uses our radially parallel LONBs.

But for *non-inertial primary observers*, Einstein's  $R^{\hat{0}}_{\hat{0}}$  equation and the Newtonian relative acceleration equation are both *non-linear* in the gravitational fields and *identical*, if one uses Einstein's *equivalence* of gravitational forces and fictitious forces [4].

For a superficial reader, Gauss's law in general relativity, Eq. (9), is “nothing new”. However: (1) Our law of Eq. (9) is derived rigorously, and it is *exactly linear for inertial primary observers*. We have not used the usual approximation of linearized gravity. The exact law is *non-linear for non-inertial primary observers*. (2) Our law of Eq. (9) only holds for auxiliary observers with *LONBs parallel along radial geodesics* to the LONBs of the primary observer at a given time. (3) Our law of Eq. (9) does not hold for the Local Inertial Frame (LIF) and the Local Inertial Coordinate Systems (LICS) around  $P_0$  (used in textbooks), where the basis vectors are parallel along geodesics radiating out from one point  $P_0$  in all spacetime directions. Our law of Eq. (9) cannot hold in a LIF, because (with curvature) LONBs cannot be parallel on all three sides of the (geodesic) triangle: (i) from  $P_0$  along the worldline of the primary inertial observer, (ii) from  $P_0$  along the worldline of an inertial particle with nonzero velocity relative to the primary observer, (iii) from the primary to the auxiliary observer at a fixed time  $t = t_0 + \delta t$ . (4) Our law of Eq. (9) only holds for our *exact* operational definition of  $\vec{E}_g$  in Eq. (4), which is probably new.

The *third step*, the derivation of Einstein's equations starting from Einstein's  $R^{\hat{0}}_{\hat{0}}$  equation for nonrelativistic sources, Eq. (9), is well known and described in textbooks: one uses local *Lorentz covariance* and *energy-momentum conservation* combined with the *contracted Bianchi identity*. This completes our rigorous and simple derivation of Einstein's field equations of general relativity,

$$G^{\hat{a}\hat{b}} = 8\pi G_N T^{\hat{a}\hat{b}}. \quad (10)$$

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